3. (6 marks)

(a) Simplify

$$\frac{log_{10}\left(4\times 3^2\right)-log_{10}\left(3\times 6\right)-3log_{10}2}{-2log_{10}2}$$

(3)

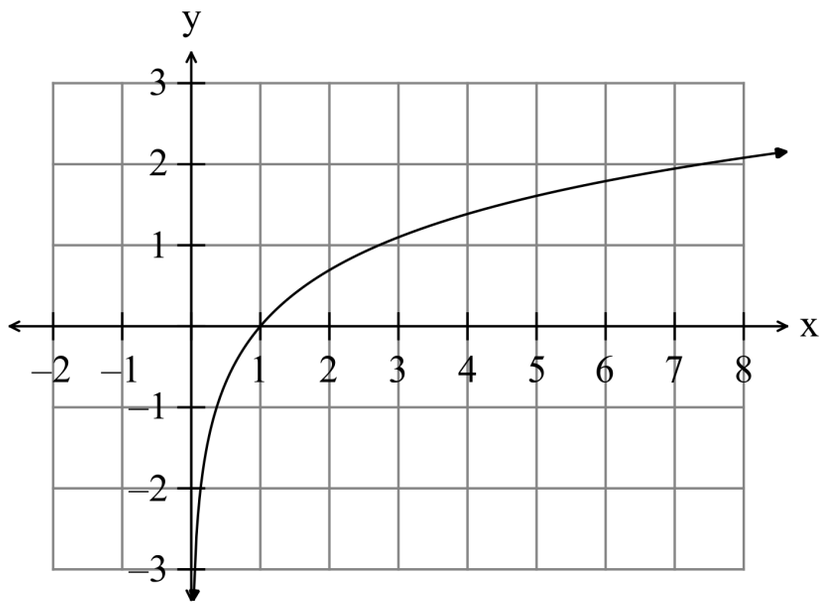
(b) Find x given

$$\left(log_3\left(x\right)-1\right)\left(ln\left(x\right)-1\right)=0$$

(3)

4. (6 marks)

(a) The graph of $y = ln(x)$ is shown on the set of axes below,



(i) Sketch on the same set of axes

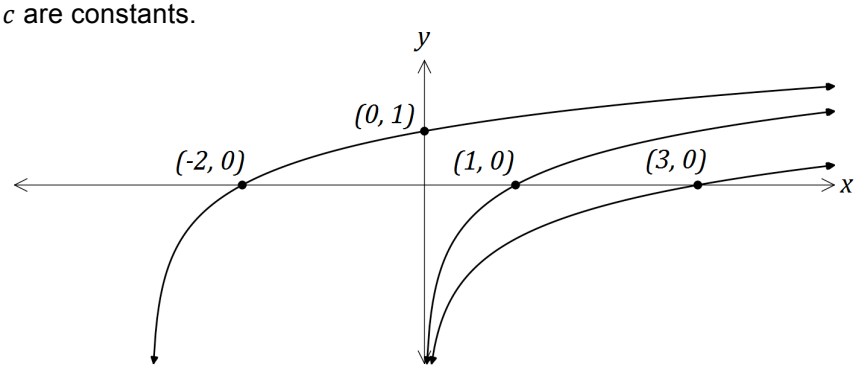
$y = 2ln(x)$ and $y = 1 - ln(x)$ (2+2)

(ii) Find the inverse of f given $f(x) = ln(x)$.
State the domain of the inverse. (2)

Question 3 **(7 marks)**

(a) The function f is defined by $f(x) = \log_a x$, $x > 0$, where a is a constant, $a > 1$.

The graphs shown below have equations $y = f(x)$, $y = f(x + b)$ and $y = f(x) + c$, where b and c are constants.



Determine the values of the constants a, b and c. (4 marks)

(b) Determine

(i) the equation of the asymptote of the graph of $y = \log_e(x - 3) - 2$. (1 mark)

(ii) the coordinates of the y-intercept of the graph of $y = \log_2(x + 8) - 5$. (2 marks)

9. (6 marks)

(Consider the functions $y = f(x)$, $y = f'(x)$ and $y = f''(x)$ graphed below.

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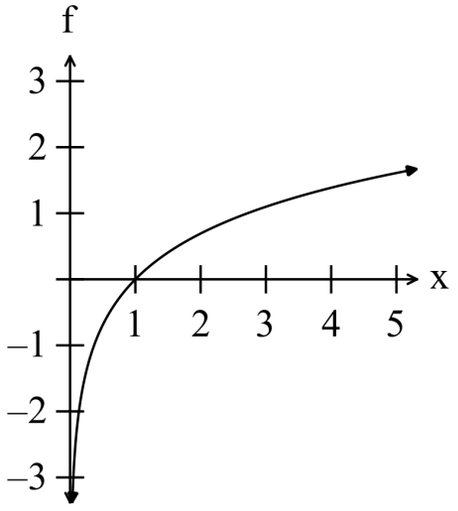

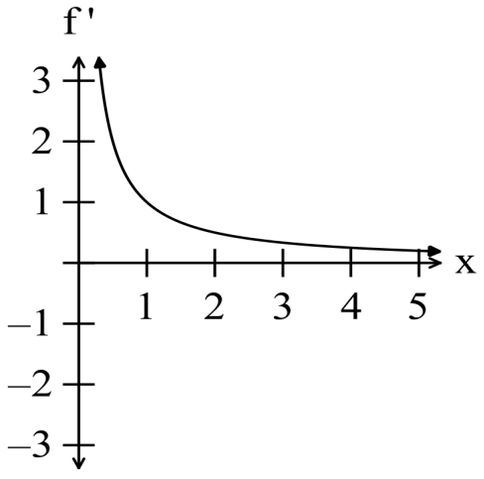

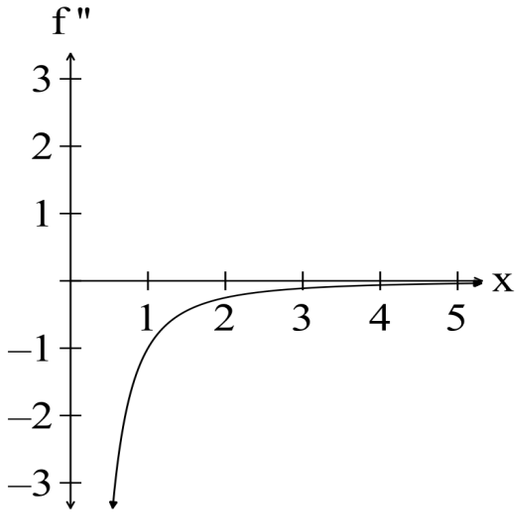

(a) Use the graphs of $y = f(x)$, $y = f'(x)$ and $y = f''(x)$ to explain why there are no turning points and no points of inflection on the graph of $y = f(x)$. (3)

(b) Comment on the concavity of the graph of $y = f(x)$ with reference to $y = f''(x)$. (2)

(c) Determine the equation for each of the three graphs. (3)

Question 19 **(7 marks)**

The moment magnitude scale M_w is used by seismologists to measure the size of earthquakes in terms of the energy released. It was developed to succeed the 1930's-era Richter magnitude scale.

The moment magnitude has no units and is defined as $M_w = \frac{2}{3}\log_{10}(M_0) - 10.7$, where M_0 is the total amount of energy that is transformed during an earthquake, measured in dyn·cm.

(a) On 28 June 2016, an estimated 2.82×10^{21} dyn·cm of energy was transformed during an earthquake near Norseman, WA. Calculate the moment magnitude for this earthquake. (1 mark)

(b) A few days later, on 8 July 2016, there was another earthquake with moment magnitude 5.2 just north of Norseman. Calculate how much energy was transformed during this earthquake. (2 marks)

(c) Show that an increase of 2 on the moment magnitude scale corresponds to the transformation of 1000 times more energy during an earthquake. (4 marks)

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3. (6 marks)

(a) $\dfrac{log_{10}\left(4\times3^2\right)-log_{10}\left(3\times6\right)-3log_{10}2}{-2log_{10}2}$

$=\dfrac{log_{10}\left(\dfrac{4\times3^2}{3\times6\times8}\right)}{-2log_{10}2}$ ✓

$=\dfrac{log_{10}\left(\dfrac{1}{4}\right)}{-2log_{10}2}$ ✓

$=\dfrac{-2log_{10}2}{-2log_{10}2}$

$=1$ ✓

(b) $\left(log_3\left(x\right)-1\right)\left(ln\left(x\right)-1\right)=0$

$log_3\left(x\right)-1=0 \quad or \quad ln\left(x\right)-1=0$ ✓

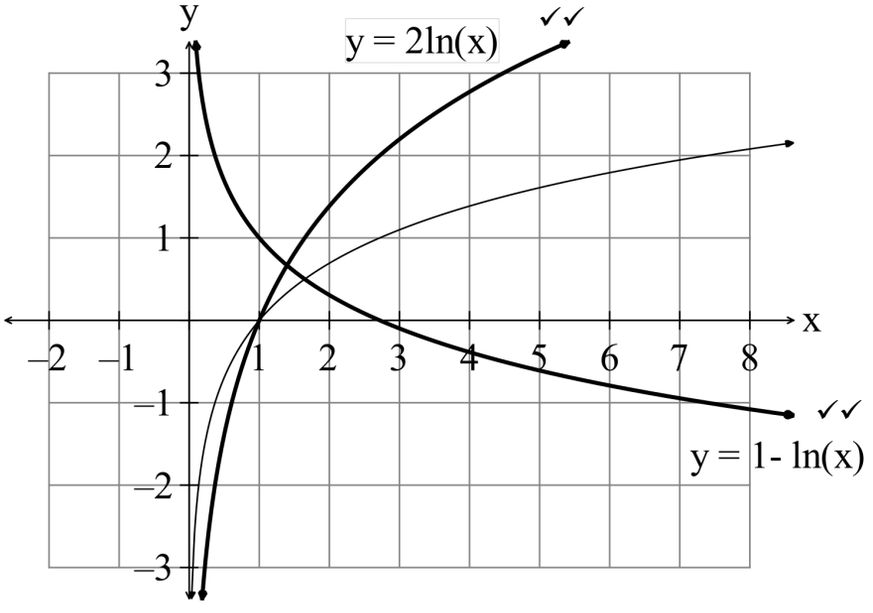
$log_3\left(x\right)=1 \quad or \quad ln\left(x\right)=1$

$x=3 \quad or \quad x=e$

✓ ✓

4. (6 marks)

(a) (i)



✓✓ ✓✓

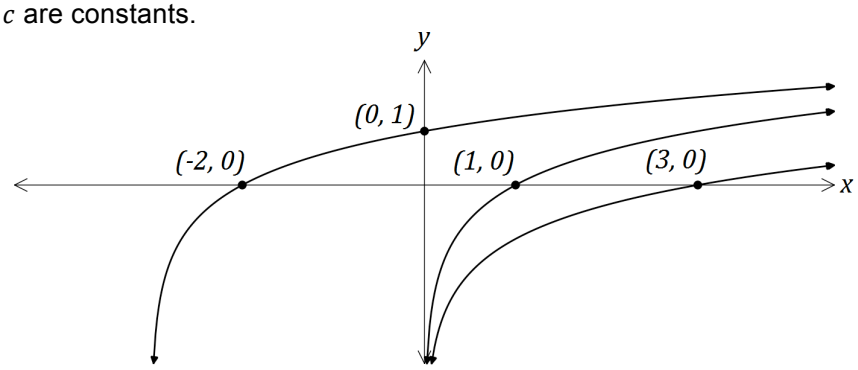
(ii) $f\left(x\right)=ln\left(x\right) \Rightarrow f^{-1}\left(x\right)=e^x \quad for\ x\in Re$

✓ ✓

Question 3 **(7 marks)**

(a) The function f is defined by $f(x) = \log_a x$, $x > 0$, where a is a constant, $a > 1$.

The graphs shown below have equations $y = f(x)$, $y = f(x + b)$ and $y = f(x) + c$, where b and c are constants.



Determine the values of the constants a, b and c. (4 marks)

Solution
$f(x + b)$ is only function that could pass through $(-2, 0)$. Hence $0 = f(-2 + b)$ and so $b = 3$. Using $(0, 1)$, $1 = \log_a(0 + 3) \Rightarrow a = 3$ $\log_3 1 = 0$ and so $f(x)$ must pass through $(1, 0)$ $f(x) + c$ passes through $(3, 0) \Rightarrow 0 = \log_3 3 + c = 0$ and so $c = -1$
Specific behaviours
✓ starts by using $f(x + b)$ and $(-2, 0)$ ✓ determines b ✓ determines a ✓ determines c

(b) Determine

(i) the equation of the asymptote of the graph of $y = \log_e(x - 3) - 2$. (1 mark)

Solution
$x = 3$
Specific behaviours
✓ writes asymptote as equation

(ii) the coordinates of the y-intercept of the graph of $y = \log_2(x + 8) - 5$. (2 marks)

Solution
$\log_2(8) - 5 = 3\log_2 2 - 5 = -2$ At $(0, -2)$
Specific behaviours
✓ substitutes and simplifies ✓ writes using coordinates

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8. (7 marks)

(a) (i) $f(t)=\sqrt{sin(\pi t)}$

$f'(t)=\frac{1}{2}\left(sin(\pi t)\right)^{\frac{-1}{2}}\pi cos(\pi t)$ ✓ ✓

$f'(t)=\frac{\pi cos(\pi t)}{2\sqrt{sin(\pi t)}}$ ✓

(ii) $\int_{\frac{1}{6}}^{\frac{1}{2}}\frac{\pi cos(\pi t)}{2\sqrt{sin(\pi t)}}dt=\left[\sqrt{sin(\pi t)}\right]_{\frac{1}{6}}^{\frac{1}{2}}$ ✓

$=\sqrt{sin\left(\frac{\pi}{2}\right)}-\sqrt{sin\left(\frac{\pi}{6}\right)}$

$=1-\frac{1}{\sqrt{2}}$ ✓

(b) $F(x)=\frac{d}{dx}\int_1^x\left(\frac{1}{t}\right)dt=\frac{1}{x}$ ✓

$\int_1^2 F(x)\,dx=\int_1^2\frac{1}{x}dx=\left[ln(x)\right]_1^2=ln(2)-ln(1)=ln(2)$ ✓

9. (8 marks)

(a) Turning points occur when $f'(x)=0.$ ✓

There are no points where this occurs so there are no turning points. ✓

Likewise, there are no points where $f''(x)=0,$ so there are no points of inflection. ✓

(b) $y=f''(x)<0$ which suggests that the concavity is concave downwards for all x values. ✓✓

(c) $f(x)=ln(x),\quad f'(x)=\frac{1}{x},\quad f''(x)=\frac{-1}{x^2}$ ✓ ✓ ✓

Question 19 **(7 marks)**

The moment magnitude scale M_w is used by seismologists to measure the size of earthquakes in terms of the energy released. It was developed to succeed the 1930's-era Richter magnitude scale.

The moment magnitude has no units and is defined as $M_w = \frac{2}{3}\log_{10}(M_0) - 10.7$, where M_0 is the total amount of energy that is transformed during an earthquake, measured in dyn·cm.

(a) On 28 June 2016, an estimated 2.82×10^{21} dyn·cm of energy was transformed during an earthquake near Norseman, WA. Calculate the moment magnitude for this earthquake. (1 mark)

Solution
$M_w = 3.6$
Specific behaviours
✓ calculates MM

(b) A few days later, on 8 July 2016, there was another earthquake with moment magnitude 5.2 just north of Norseman. Calculate how much energy was transformed during this earthquake. (2 marks)

Solution
$5.2 = \frac{2}{3}\log_{10} x - 10.7$ $x = 7.08\times10^{23}\ \text{dyn}\cdot\text{cm}$
Specific behaviours
✓ substitutes ✓ solve for energy

(c) Show that an increase of 2 on the moment magnitude scale corresponds to the transformation of 1000 times more energy during an earthquake. (4 marks)

Solution
$M_w = \frac{2}{3}\log_{10}(x) - 10.7 \ldots (1)$ and $M_w + 2 = \frac{2}{3}\log_{10}(y) - 10.7 \ldots (2)$ $(2) - (1)\text{: } 2 = \frac{2}{3}(\log_{10} y - \log_{10} x)$ $\log_{10}\frac{y}{x} = 3$ $\frac{y}{x} = 10^3 = 1000\ \text{times greater}$
Specific behaviours
✓ writes two equations for M and $M + 2$ ✓ subtracts equations ✓ uses log laws to simplify ✓ converts to exponential form and simplifies *NB Max ✓✓ if uses specific values rather than general case*

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